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Contributions to the muon anomalous magnetic moment are evaluated in the technicolor model with scalars and topcolor assisted technicolor model. We found that both models can give the large contributions required by the E821 experiment. However, this occurs only for a very large f/f' in the technicolor model with scalar, which is strongly disfavored by the experiment of $b \rightarrow s\gamma$. As for the top-color assisted technicolor model, it happens in case of a large $\tan\theta'$, which is not in accord with the motivation for building this model. So, if the E821 experiment result persists, it would be a challenge to technicolor models.

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Introduction

While it is often argued that the standard model (SM) should be augmented by new physics at higher energy scales because of some unanswered fundamental questions, the recently reported 2.6 standard deviation of the muon anomalous magnetic moment over its SM prediction [1] may serve as the first evidence of existence of new physics at a scale not far above the weak scale. Since the experiment of the muon anomalous magnetic moment will be further developed, it will be a very powerful tool for testing the SM and probing new physics.

There are numerous speculations on the possible forms of new physics, among which supersymmetry (SUSY) and technicolor are the two typical different frameworks. Although both frameworks are well motivated, SUSY has been more favored by precision electroweak experiment than technicolor. The SUSY contributions to the muon anomalous magnetic moment were computed by many authors [2,3]. It was found that SUSY can give the large contributions and thus naturally explain the reported deviation [3]¹.

Confronted with the new experiment results of the muon anomalous magnetic moment, both supersymmetry and technicolor [5,6] should be examined. In this letter, we will evaluate technicolor contributions. First we

will present a detailed analysis in the framework of technicolor model with scalars [7,8], which is likely to give significant contributions since the couplings of scalars to muon could be significantly enhanced by the parameter f/f' . Then we give an analysis for the topcolor-assisted technicolor model [9–11], which also seemingly can give large contributions since this model predicts a new gauge boson Z' . Finally, for other technicolor models, we will give a comment that they are unlikely to give the required contributions.

Technicolor with scalars

Technicolor with scalars [7,8] has a minimal $SU(N)$ technicolor sector, consisting of two techniflavors p and m . The technifermions transform as singlet under color and as fundamentals under the $SU(N)$ technicolor group. In addition to the above particle spectrum, there exists a scalar doublet ϕ to which both the ordinary fermions and technifermions are coupled. Unlike the SM Higgs doublet, ϕ does not cause electroweak symmetry breaking but obtains a non-zero effective vacuum expectation value (VEV) when technicolor breaks the symmetry.

If we write the matrix form of the scalar doublet as

$$\Phi = \begin{bmatrix} \bar{\phi}^0 & \phi^+ \\ -\phi^- & \phi^0 \end{bmatrix} \equiv \frac{(\sigma + f')}{\sqrt{2}} \Sigma', \quad (1)$$

and adopt the conventional non-linear representation $\Sigma = \exp(\frac{2i\Pi}{f})$ and $\Sigma' = \exp(\frac{2i\Pi'}{f'})$ for technipions, with fields in Π and Π' representing the pseudoscalar bound states of the technifermions p and m , then the kinetic terms for the scalar fields are given by

$$\begin{aligned} \mathcal{L}_{K.E.} = & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4} f^2 \text{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) \\ & + \frac{1}{4} (\sigma + f')^2 \text{Tr}(D_\mu \Sigma'^\dagger D^\mu \Sigma'). \end{aligned} \quad (2)$$

Here D^μ denotes the $SU(2)_L \times SU(2)_R$ covariant derivative, σ is an isosinglet scalar field, f and f' are the technipion decay constant and the effective VEV, respectively.

The mixing between Π and Π' gives

$$\pi_a = \frac{f\Pi + f'\Pi'}{\sqrt{f^2 + f'^2}}, \quad (3)$$

$$\pi_p = \frac{-f'\Pi + f\Pi'}{\sqrt{f^2 + f'^2}}, \quad (4)$$

with π_a becoming the longitudinal component of the W and Z , and π_p remaining in the low-energy theory as an

¹There are also some attempts to explain the reported deviation in other approaches [4].

isotriplet of physical scalars. From Eq. (2) one can obtain the correct gauge boson masses providing that $f^2 + f'^2 = v^2$ with the electroweak scale $v = 246 \text{ GeV}$.

Additionally, the contributions to scalar potential generated by the technicolor interactions should be included in this model. The simplest term one can construct is

$$\mathcal{L}_T = c_1 4\pi f^3 \text{Tr} \left[\Phi \begin{pmatrix} h_+ & 0 \\ 0 & h_- \end{pmatrix} \Sigma^\dagger \right] + h.c., \quad (5)$$

where c_1 is a coefficient of order unity, h_+ and h_- are the Yukawa couplings of scalars to p and m . From Eq. (5) the mass of the charged scalar at lowest order is obtained as

$$m_{\pi_p}^2 = 2\sqrt{2}(4\pi f/f')v^2 h \quad (6)$$

with $h = (h_+ + h_-)/2$. When the largest Coleman-Weinberg corrections for the σ field are included in the effective chiral Lagrangian [8], one obtains the constraint

$$\tilde{M}_\phi^2 f' + \frac{\tilde{\lambda}}{2} f'^3 = 8\sqrt{2}\pi c_1 h f^3 \quad (7)$$

and the isoscalar mass as

$$m_\sigma^2 = \tilde{M}_\phi^2 + \frac{2}{3\pi^2} [6(\frac{m_t}{f'})^4 + N h^4] f'^2 \quad (8)$$

in limit (i) where the shifted ϕ^4 coupling $\tilde{\lambda}$ is small and can be neglected [7], and

$$m_\sigma^2 = \frac{3}{2} \tilde{\lambda} f'^2 - \frac{1}{4\pi^2} [6(\frac{m_t}{f'})^4 + N h^4] f'^2 \quad (9)$$

in limit (ii) where the shifted mass of the scalar doublet ϕ , \tilde{M}_ϕ is small and can be neglected [8].

The advantage of this model is that it can successfully account for fermion masses without generating large flavor-changing neutral current effects, and without exceeding the experimental bounds on oblique electroweak radiative corrections. Furthermore, we stress at the lowest order, only two independent parameters (f/f' , m_{π_p}) in the limits (i) and (ii) mentioned above are needed to describe the phenomenology.

The contributions to the muon anomalous magnetic moment stem from the diagrams shown in Fig. 1. The relevant interactions can be extracted from Eq. (2) and Eq. (5) [12]

$$\begin{aligned} \mathcal{L} = & \left(\frac{v}{f'}\right) \frac{gm_\mu}{2m_W} \sigma \bar{\mu} \mu - \left(\frac{f}{f'}\right) \frac{igm_\mu}{2m_W} \pi_p^0 \bar{\mu} \gamma_5 \mu \\ & + \left(\frac{f}{f'}\right) \frac{igm_\mu}{2\sqrt{2}m_W} [\pi_p^+ \bar{\nu}_\mu (1 + \gamma_5) \mu - \pi_p^- \bar{\mu} (1 - \gamma_5) \nu_\mu] \\ & - ie A_\mu \pi_p^+ \overleftrightarrow{\partial}^\mu \pi_p^- . \end{aligned} \quad (10)$$

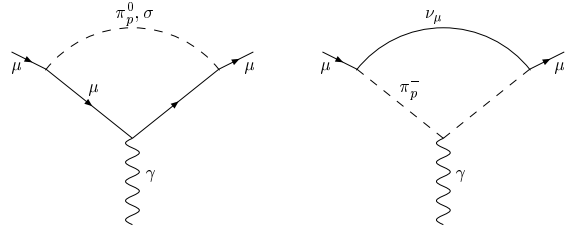


FIG. 1. Feynman diagrams for the contributions of scalars in technicolor with scalars.

Describing the $\gamma\mu\mu$ vertex in the most general Lorentz structure form [13]

$$\begin{aligned} \Gamma_\alpha = & i \{ \gamma_\alpha [F_V - F_A \gamma_5] + [iF_S + F_P \gamma_5] p_\alpha \\ & + [iF_M + F_E \gamma_5] \sigma_{\alpha\beta} p^\beta \} \end{aligned} \quad (11)$$

with p being the incoming momentum of the photon and the form factors F_i being functions of the invariant $s = p^2$, we obtain the explicit expression of the usual anomalous magnetic dipole moment as

$$\begin{aligned} a_\mu \equiv & \frac{2m_f}{e} F_M(0) = a_\mu^{\pi^0} + a_\mu^\sigma + a_\mu^{\pi^\pm} \\ = & \frac{G_F m_\mu^2}{\sqrt{2}\pi^2} \left(\frac{f}{f'}\right)^2 \left\{ f_1(a_{\pi_p}, b_{\pi_p}) + i \frac{m_\mu^2}{m_{\pi_p}} f_1(a_{\pi_p}, b_{\pi_p}) \right. \\ & \left. + \left(\frac{v}{f'}\right)^2 [f_1(a_\sigma, b_\sigma) + 2f_2(a_\sigma, b_\sigma)] \right\}. \end{aligned} \quad (12)$$

Here a_h, b_h ($h = \sigma, \pi_p$) are the roots of equation $m_\mu^2 x^2 - m_h^2 x + m_h^2 = 0$ with the convention $a_h > b_h$, and the functions f_i are given by

$$\begin{aligned} f_1(x, y) = & \frac{1}{x-y} \left[y^3 \ln \frac{y}{y-1} - x^3 \ln \frac{x}{x-1} \right] + x + y + \frac{1}{2}, \\ f_2(x, y) = & \frac{1}{x-y} \left[x^2 \ln \frac{x}{x-1} - y^2 \ln \frac{y}{y-1} \right] - 1. \end{aligned} \quad (13)$$

We should bear in mind that the parameter space chosen in our numerical calculations is consistent with known experimental measurements. At first, we apply the lower experimental bound of 107.7 GeV for the SM Higgs and 78.6 GeV for charged Higgs boson in Two-Higgs-Doublet model [14] directly to constrain the mass of the scalars, and set the mass of the charged scalar less than 1 TeV , leaving f/f' as a free parameter. We perform a complete scan of the parameter space of technicolor with scalars, and display the additional contribution due to the neutral scalars as a function of f/f' in Fig. 2, with the mass of m_{π_p} varying from 107.7 GeV to 1 TeV for any fixed value of f/f' .

A couple of remarks are due regarding our results:

1. In limit (i), contributions to the muon anomalous magnetic moment from the neutral scalars can be

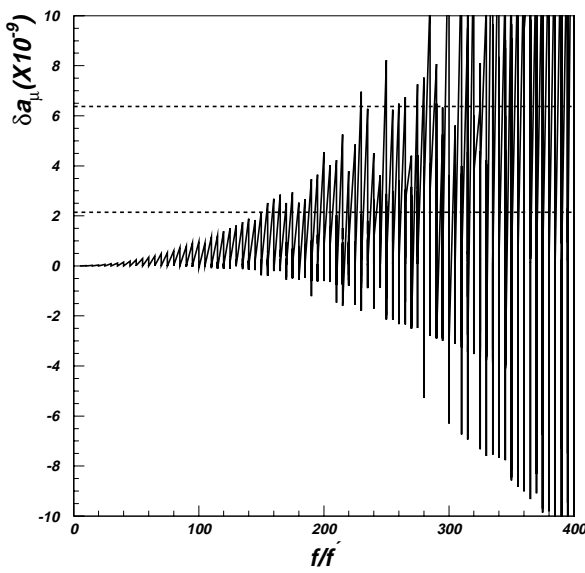


FIG. 2. The contributions of the neutral scalars to the muon anomalous magnetic moment as a function of f/f' in the technicolor model with scalars. For any fixed value of f/f' , the mass of m_{π_p} is varied from 107.7 GeV to 1 TeV. The dashed lines denote the current lower and upper experimental bounds on new physics contributions at 90% C. L..

positive or negative and not large enough except for very large f/f' ; whereas the charged scalar can only provide the contribution to the imaginary part which is much smaller than the real one.

2. Unlike the case in limit (i), the additional contributions are negligible in limit (ii). No very large f/f' is allowed by the model.
3. As expected, for any fixed value of f/f' , the total contribution decreases rapidly as the mass of the charged scalar increases.

The limit on f/f' has been investigated by several authors, and obtained from the studies of $b \rightarrow X_c \tau \nu$ [15], $b \rightarrow X_s \gamma$ [15,16], $Z \rightarrow b\bar{b}$ [16], $B \rightarrow X_s \mu^+ \mu^-$ [17] and $B-\bar{B}$ mixing [7,8,16], which is given by [15]

$$\frac{f}{f'} \leq 0.03 \left(\frac{m_{\pi_p}}{1 \text{ GeV}} \right) \quad (95\% \text{ C. L.}). \quad (14)$$

According to this limit, $f/f' > 200$, which is needed to give large contributions to the muon anomalous moment, seems unlikely.

Topcolor-assisted technicolor model

The other competitive candidate, which might provide large additional contributions to the muon anomalous moment, is the topcolor-assisted technicolor model [9–11]. The model assume: (i) electroweak interactions are broken by technicolor; (ii) the top quark mass is large because it is the combination of a dynamical condensate component, generated by a new strong dynamics,

together with a small fundamental component, generated by an extended technicolor [6,18]; (iii) the new strong dynamics is assumed to be chiral critically strong but spontaneously broken by technicolor at the scale $\sim 1 \text{ TeV}$, and it generally couples preferentially to the third generation. This needs a new class of technicolor models incorporating “top-color”. The dynamics at $\sim 1 \text{ TeV}$ scale involves the gauge structure:

$$SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \\ \rightarrow SU(3)_{QCD} \times U(1)_{EM}$$

where $SU(3)_1 \times U(1)_{Y_1}$ [$SU(3)_2 \times U(1)_{Y_2}$] generally couples preferentially to the third (first and second) generation, and is assumed to be strong enough to form chiral $\langle \bar{t}t \rangle$ but not $\langle \bar{b}b \rangle$ condensation by the $U(1)_{Y_1}$ coupling. A residual global symmetry $SU(3)' \times U(1)'$ implies the existence of a massive color-singlet heavy Z' and an octet B_μ^A . A symmetry-breaking pattern outlined above will generically give rise to three top-pions, $\tilde{\pi}$, near the top mass scale.

In this model, the muon anomalous magnetic moment receives additional contributions only from a gauge boson Z'_μ and top-pions $\tilde{\pi}^0$, $\tilde{\pi}^\pm$, with Feynman diagrams similar to Fig. 1 and σ , π_p replaced by Z'_μ , top-pions, respectively. The relevant Lagrangian is given by [10]

$$\mathcal{L}^{eff} = \frac{1}{2} g_1 \tan^2 \theta' Z'_\alpha [\bar{\mu}_L \gamma^\alpha \mu_L + 2 \bar{\mu}_R \gamma^\alpha \mu_R] \\ + \frac{m_\mu^*}{f_\pi} \left[\frac{i}{\sqrt{2}} \bar{\mu}_L \tilde{\pi}^0 \mu_R + \bar{\nu}_{\mu L} \tilde{\pi}^+ \mu_R + h.c. \right] \quad (15)$$

where g_1 is the $U(1)_Y$ coupling constant at the scalar $\sim 1 \text{ TeV}$. The SM $U(1)_Y$ and the $U(1)'$ field Z'_α is then defined by orthogonal rotation with mixing angle θ' . $m_\mu^* \approx m_\mu$ indicates the piece of the muon mass generated by instanton effects and $f_\pi \approx 50 \text{ GeV}$ is the top-pions decay constant.

When we use the Lagrangian in Eq. (15) to calculate the additional technicolor effects to the muon anomalous magnetic moment, the small contribution from the charged top-pions, as well as neutral one, can be neglected safely, as in the case of technicolor with scalar. Now we present the contributions from the neutral gauge boson Z'

$$a_\mu = \frac{g_1^2}{32\pi^2} \tan^4 \theta' \left\{ -a_{Z'} - b_{Z'} - \frac{11}{2} \right. \\ + \frac{1}{a_{Z'} - b_{Z'}} \left[a_{Z'} (a_{Z'}^2 + 5a_{Z'} - 6) \ln \frac{a_{Z'}}{a_{Z'} - 1} \right. \\ \left. \left. - b_{Z'} (b_{Z'}^2 + 5b_{Z'} - 6) \ln \frac{b_{Z'}}{b_{Z'} - 1} \right] \right\}. \quad (16)$$

The additional contribution due to the neutral gauge boson Z' to the muon anomalous magnetic moment as a function of $\tan \theta'$ is shown in Fig. 3, with the mass of

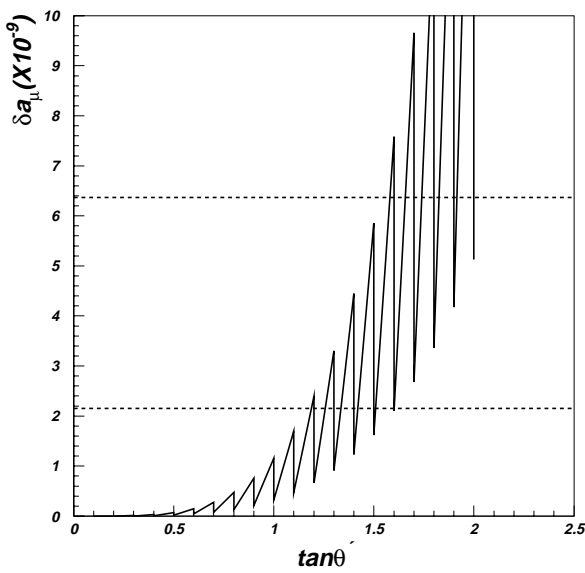


FIG. 3. Additional contribution due to the neutral gauge boson Z' to the muon anomalous magnetic moment as a function of $\tan \theta'$. For any fixed value of $\tan \theta'$, the mass of Z' is varied from 100 GeV to 1 TeV. The dashed lines are the same as in Fig. 2.

Z' varying from 100 GeV to 1 TeV for any fixed value of $\tan \theta'$. One can see the topcolor-assisted model can also give large contributions in case of a large $\tan \theta'$, which, however, is not favored by the model because a small $\tan \theta' \ll 1$ is ultimately demanded to select the top quark direction for condensation.

We also scanned other technicolor models and found they are more unlikely to give the large contributions to the muon anomalous magnetic moment since the new particle couplings to the muon do not have any enhancement factors like f/f' .

In summary, we evaluated technicolor contributions to the muon anomalous magnetic moment in two frameworks of technicolor: technicolor model with scalars and the topcolor-assisted technicolor model. We found that the technicolor model with scalars can give the large contributions required by the E821 experiment only for a very large f/f' which, however, is strongly disfavored by the experiment of $b \rightarrow s\gamma$. The topcolor-assisted model can also give large contributions in case of a large $\tan \theta'$, which is not in accord with the motivation for building this model. Therefore, if the current deviation of the muon anomalous moment from its SM prediction persists as the experiment is further developed, it would be a challenge to the technicolor models.

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